## "Restricted Choice" In Bridge and Other Related Puzzles

P. Tobias, 9/4/2015

Before seeing how the principle of Restricted Choice can help us play suit combinations better let's look at the best way (in order to succeed most often) to play 4 standard holdings:
KJ753
KJ753
AQ10843
AKQ10
1)
2)

A862
3)

765
4)
752

AQ104
or

A86
Note: Throughout this discussion we assume declarer has no information about suit distributions from the bidding or play that might change the way suits are likely to split.

In 1) we play the ace and next plan to finesse the $J$ while in 2 ) we play the $A$ and next play to play the K (remember the old and correct here) adage about Q finesses - 5 ever, 4 never).

In 3) the best play for all the tricks is low to the Q. If, however, that loses to the K and the J does not appear when you lead next, still go up with the A, playing East for KJ doubleton. In 4), play the $\mathrm{A}, \mathrm{K}$ and then the Q instead of finessing the 10.

Note that in 2), 3) and 4) at the point where you have to decide whether to finesse or play for the drop there is one card left outstanding after West plays and you can (correctly) reason that East has one more un-played card in her hand to hold that missing honor and therefore playing for the drop is the best chance play.

K10953
Now look at 5)
and assume you play the A and East plays either the J or the Q .
A842
Does this change your mind after you lead towards the K1095 and West plays the 6? It looks a lot like 2) and, again, East has more room left to hold the missing honor. Next look at

AJ10953
6)
and assume you lead to the J and East plays the K or Q .
842
When you next lead to the dummy, after West plays small, only the other honor is left outstanding and, again, East has more room left to hold it. But, as we shall next see, RESTRICTED CHOICE surprisingly tells us to finesse for the missing honor in both 5) and 6) and this can improve our odds of success by as much as a factor of 2!

# Statement of the Principle of Restricted Choice (Taken From Jeff Rubens (1964): The play of a card which may have been selected as a choice of equal plays increases the chances that his choice was restricted. 

In other words, your best bet is to assume the defender didn't have a choice and had to play the card he played. If he could have had both the $Q$ and $J$, assume he had only the one he played. If he could have had the K and Q , again assume he had only the one he played. The same is true for lower spot cards in similar situations, and also (as we will see later) for even opening leads when there could have been a choice of equivalent suits to lead.

Let's go back to 5) and 6). In 5), the defender either had to play the singleton honor (Q or J) or had a choice of which to play. "Restricted Choice" says assume the singleton honor and finesse East for the other honor. In 6), the same holds for the missing K or Q and following "Restricted Choice" you should finesse a second time.

Why this is right is a sophisticated exercise in probability theory involving Bayes Theorem and combinatorial calculations. All I will say here is that it is correct and it will work better than any other way of playing these suit combinations and will, in many situations, almost double your odds of success. It does assume, however, that any defender with both touching honors will choose which to play randomly - as this is the best strategy for a defender (if you always play the Q , for example, holding $\mathrm{Q} \& \mathrm{~J}$ doubleton, then when you play the J declarer has a $100 \%$ certain finesse!). If you want to see the actual probability discussions, google Restricted Choice in Bridge to view more than you ever wanted to know.

Eddie Kantor stated the principle nicely as follows: When the opponents hold 2 equally important cards and one has appeared on the previous trick, then take the finesse for the remaining important card.

Let's try the principle on the next 3 examples.
A972
AQ94
Q42
7)
8)
9)

KQ3
K62
AK85

In 7) assume you play the K followed by the Q and East plays either the 10 or the J when following to the Q . In 8) you play the A and K and East again plays the 10 or the J on the second lead. Finally, in 9) you play the A and the $Q$ and see West play 2 of the three cards higher than the 8 (i.e the J 10 , J 9 or 109). On the third lead, do you finesse or play for the drop?

An easy choice following Kantor or Rubens - you use restricted choice and assume the defender had to play the cards observed and does not have the missing equal card. Therefore you finesse in each of these situations and significantly improve your chances over playing for the drop.

But watch out for the following holding:

AKQ942
10) or, equivalently

63

KQ9642

A5

In both cases you play the A and East plays the 10 or J. Does Restricted Choice (R.C.) apply? Should you finesse the next time you lead the suit?

The answer is NO, if East is an experienced player who knows it can't hurt to False Card with the J or 10 holding J10x. R. C. does not apply when East might very likely be false-carding. Only if you think East would never make that kind of false card, should you follow R. C. and finesse. Note: When on defense, think about situations like this and false card to give declarer a chance to go wrong.

Are there other applications of R. C. we can look at? Yes, many - these come from the excellent write-up on R. C. in the Encyclopedia of BRIDGE.

432

K109
12)
12)

432
13)

J94

Q32

2
14)

QJ87654

QJ9
11)

Play these yourself, using R, C, principles. In 11) you lead to the $Q$ and lose to the $A$ or $K$. What do you play the next time you lead to the J9? In 12) you lead to the 10 and lose to the J or Q. Again, what do you play the next time you lead to dummy? In 13) you lead to the $Q$ and lose to
the K or A and lead next to the J9. Finally, in 14) you lead to the $Q$ and East plays the 9 or 10 and West wins the K or A - what next? Note that 14) turns out to be a double application of R. C. on each defender when East plays a 9 or 10. (See page 6 for answers)

Here is an example from an important team match where declarer went wrong:
K42
83
K932
AK87
15)

Declarer, in 3NT, received a 5H lead. Won by East with the K and a low heart was returned. What to play?

A5 Declarer thought East might hold the AK and went up Q Q109 and lost the match because of it. R. C. says that with both the AQJ765 A and K, East could have won the first trick with either of 104 them. So, assume the choice was restricted and play West for the AH .

For the last example, an opening lead where R. C. comes into play. You are in 6S.
A432 West leads the 7D and East wins the A and leads a low
AK432 club. Do you finesse or play hearts to be 3-3, which will J10 generate enough pitches to get rid of all your clubs?
16)

KQJ1096

AQ10
A 3-3 heart Split is about $36 \%$ and a finesse is $50 \%$. So, it looks like you should finesse in in clubs. But, is there an R. C. argument that changes the odds? If West had two worthless minors to choose a safe opening lead from, he could have picked a club-R.C. applies and assume he has the KC and had to lead a diamond - so try hearts!

## The Monty Hall Controversy and Restricted Choice

The controversy began when Marilyn vos Savant published a puzzle in her Parade Magazine column in June of 1990. One of her readers posed the following question: "Suppose you're on a game show (Like "Let's Make a Deal'), and you're given a choice of three doors: Behind one door is a car; behind the others, goats. You pick a door, say number 1, and the host (Monty Hall), who knows what's behind the doors, opens another door, say number 3, which has a goat. He says to you, 'Do you want to pick door number 2 or stick with number 1?' Is it to your advantage to switch your choice of doors?" The audience is yelling "Switch" or "Stay with your choice" and you have to decide.

Ms. Savant, answered "Yes - you improve your chances of getting the car by a factor of 2 to 1." Because of the estimated 10,000 letters she received in response, many strongly disputing her answer, she published a second article on the subject. Of the disagreeing letters, more than 1000 were from Phd's and mathematicians or statisticians.

Due to the fervor created by Ms. Savant's two columns, the New York Times published a large front page article in a 1991 Sunday issue which declared: "Her answer... has been debated in the halls of the C.I.A. and the barracks of fighter pilots in the Persian Gulf. It has been analyzed by mathematicians at M.I.T. and computer programmers at Los Alamos National Laboratory in New Mexico. It has been tested in classes ranging from second grade to graduate level at more than 1,000 schools across the country."

The Monty Hall problem "has immortalized both Marilyn vos Savant and Let's Make a Deal ... It had to come as a surprise to the show's creators that after airing 4,500 episodes in nearly twenty-seven years, it was this question of mathematical probability that would be their principal legacy."

The CBS drama series NUMB3RS featured the Monty Hall Problem in the final episode of its 20042005 season. The show's mathematician offered his own, very definite solution to the problem involving hidden cars and goats.

The 2008 movie 21 opens with an M.I.T. math professor (played by Kevin Spacey) using the Monty Hall Problem to explain mathematical theories to his students. His lecture also includes the popular "goats and cars behind three doors" example favored by many versions of the Problem.

The London FINANCIAL TIMES published a column about the Monty Hall Problem on August 16, 2005, declaring positively that "the answer is, indeed, yes: you should change." However, the columnist, John Kay, noted that "Paul Erdos, the great mathematician, reputedly died still musing on the Monty Hall problem." The column resulted in several letters published on the "Leaders and Letters" page of the FINANCIAL TIMES on August 18 and 22 - and two follow-up columns by Mr. Kay on August 23 (So you think you know the odds) and August 31 (The Monty Hall problem - a summing up) in which he acknowledges that he received "a large correspondence on Monty Hall." Numerous other articles and books have been published featuring this problem.

The false but intuitive appealing solution goes like this: Monty has not given you any new information since there will always be a door with a goat behind it for him to open. So, you might as well stick with your original door. Or, many people reason, now there are two doors left - and one of them has the car
behind it. So, your chances are 50-50 with either of these doors and you might as well stick with the original door.

The correct solution is as follows: Your chances originally are $1 / 3$ for the door you picked and that does not change. Whenever the car is in behind one of the other two doors, Monty has a "restricted choice" and will always pick the door with the goat. The other door will have the car behind it $2 / 3$ of the time. So you double your chances by switching. The "restricted choice" principle to remember is "go with the choice that comes from assuming Monty had to pick the door he did - in other words that he had no choice".

Here's a way to test the validity of this solution: select three cards from a deck, one being the ace of spades. Mix them and have someone select (face down) a card. Look at the two remaining and remove a card (not the ace) and show it. Put the other card in front of you and ask if the selector wants to bet even money he has the ace instead of you. Try it 100 times and see how often you have the ace. Try this with all 52 cards and, after someone draws a card face down, look at the cards and show 50 nonace of spades cards and put the remaining one face down in front of you. Now it should be clear who has a much better chance of having the ace (but some people will still insist the chances are $50-50$ !).

## Another Similar Problem

Three prisoners on death row are told by the warden that exactly 1 will be pardoned and the other two executed the next day. John says to the warden, "Please tell me one name out of Peter and Jesse (the other 2 prisoners) that will be executed tomorrow - one or both will, anyway, so telling me doesn't matter. Then, it will only be me or the one not named who will receive a pardon and my odds of survival increase from $1 / 3$ to $1 / 2$. I'll sleep much better knowing my chances are $50 / 50$. The warden agrees and tells him a correct non-pardoned name.

Is John correct in his assessment of his chances and how does it relate to R. C.?
Answer: R.C. implies that the warden had a restricted choice when he picked the name he tells John - so assuming the other prisoner is the one pardoned is the most likely scenario. John's chances stay at $1 / 3$ but the prisoner not named by the warden has a $2 / 3$ chance of being pardoned. (This is exactly like Monty Hall picking a door with a goat behind it, thereby increasing the likelihood of the car being behind the other door.)

Answers to 11), 12), 13) and 14): In 11). play the $J$ assuming East does not also have the other touching honor - do not finesse the 9! But do finesse the 9 in 12) because R.C. tells you not to play East for the other touching honor. In 13), R.C. says to play East for the other touching honor so you again finesse the 9. In 14), you assume East did not have the 109 doubleton when she plays one of those cards. Also, you assume West did not have both the A and K when he plays one of them. Therefore, the only hope is to play East for a doubleton high honor (not a doubleton 109).R.C. works on both defenders and tells you to play West for the 9 or 10 and East for the A or K.

